

Vorticity, Gyroscopic Precession, and Spin-Curvature Force

Wei Chieh Liang* and Si Chen Lee†

Graduate Institute of Electronics Engineering, National Taiwan University, Taipei, Taiwan

(Dated: October 24, 2012)

In investigating the relation between vorticity and gyroscopic precession, we calculate the vorticity vector in Godel, Kerr, Lewis, Schwarzschild, Minkowski metric and find out the vorticity vector of the specific observers is the angular velocity of gyroscopic precession. Furthermore, considering space-time torsion will flip the vorticity and spin-curvature force to opposite sign. This result is very similar to the behavior of positive and negative helicity of quantum spin in Stern-Gerlach force. It implies that the inclusion of torsion will lead to analogous property of quantum spin even in classical treatment.

PACS numbers: 04.50.Kd, 95.30.Sf, 04.20.Cv

I. INTRODUCTION

Frame dragging effect (Lense-Thirring effect) was a prediction of general relativity that had been verified by the Gravity Probe-B experiment [1]. The effect can be measured by the precession of gyroscope moving in gravitational field with rotating source (the earth). The rotating source will drag the inertial frame (gyroscope) and causes the gyroscopic precession with respect to the fixed stars [2, 3].

According to Smalley [4] who considered the Godel metric in Riemann-Cartan space-time, the appearance of torsion tensor induced by spinning fluid will not change the magnitude of the angular velocity of global rotation but result in the opposite sign. The spin-flip of angular velocity was claimed to be consistent with the result of Tsoubelies [5] who found that the spin density produced a global stationary space-time outside a static but spin-polarized cylinder space-time. In that case, the spin is responsible for the presence of t - ϕ cross terms (t, r, θ, ϕ are coordinates) in the metric field outside. It reflects the dragging of inertia. Thus spin has a status equal to that of orbital angular momentum.

After further consideration, it is found that the so called spin-flip is an ambiguous concept. In their model, the torsion is caused by fluid with intrinsic but classical spin. In normal Einstein-Cartan theory, torsion is associated with quantum mechanical spin. It is not clear, what is the true meaning of global rotation with opposite sign. The global angular velocity of Godel universe actually is the vorticity vector [4].

Following this idea, the vorticity vector of Kerr, Lewis, Schwarzschild, and Minkowski (rotating coordinate) metric in Riemann and Riemann-Cartan space-time were investigated to find out the deeper physical meaning of vorticity and torsion. First the basic knowledge of relativistic gyroscopic precession, Riemann-Cartan space-time, Einstein-Cartan-Sciama-Kibble (ECSK) theory are

reviewed and the meaning of vorticity is discussed. The vorticity vectors of each metric by selecting specific observer in Riemann and Riemann-Cartan space-time are calculated, then the spin-curvature force from Mathisson-Papapetrou equation of Godel metric is analyzed. Finally, the conclusions are given.

II. PRELIMINARIES

A. Gyroscopic precession

The spin angular momentum of a body will be affected by the curvature of the space-time, so a spinning gyroscope orbiting a massive body will undergo precession of its spin vector S : $dS/dt = \Omega \times S$, Ω is the angular velocity of precession. There are three relativistic contributions to this precession [1–3].

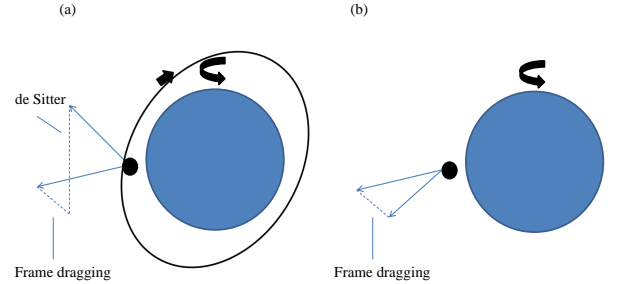


FIG. 1: (a) When the gyroscope moves around a rotating mass, it will undergo both de Sitter and frame dragging precession. (b) If the gyroscope is at static point near the rotating mass, it only feels the precession due to the dragging of the space-time.

*Electronic address: d95943030@ntu.edu.tw

†Electronic address: scllee@ntu.edu.tw

position of non-aligned Lorentz boosts, which is a special relativistic effect. 2. The de Sitter precession, associated with the gyroscopic transportation in curved space-time generates a precession with respect to a fixed reference. 3. The frame dragging precession, arising from the rotation of the central body which causes the inertial frames to be dragged along with respect to the fixed stars. It is of purely geometrical origin and is independent of the orbital elements of the gyroscope as shown in Fig.1(a) and 1(b)

B. The Riemann-Cartan space-time (U4 geometry) and ECSK theory

The most natural modification of general relativity in Riemannian geometry (V4) is ECSK theory. This geometry is no longer Riemannian but Riemann-Cartan space-time with a non-symmetric and metric compatible connection. It brings up a new geometrical property of space-time which is the torsion tensor [6]. The definition of torsion tensor $T_{\alpha\beta}^{\mu}$ is

$$T_{\alpha\beta}^{\mu} = \frac{1}{2}(\Gamma_{\alpha\beta}^{\mu} - \Gamma_{\beta\alpha}^{\mu}). \quad (1)$$

where $\Gamma_{\alpha\beta}^{\mu}$ are the connection coefficients. In Einstein-Cartan theory, the intrinsic angular momentum (particle spin) is the source of space-time torsion [6]. The connection of U4 $\Gamma_{\alpha\beta\mu}$ is different from the Levi-Civita connection in V4 $\tilde{\Gamma}_{\alpha\beta\mu}$.

$$\Gamma_{\alpha\beta\mu} = \tilde{\Gamma}_{\alpha\beta\mu} - K_{\alpha\beta\mu}, \quad (2)$$

the second term is the contortion tensor which is given by

$$K_{\alpha\beta\mu} = -T_{\alpha\beta\mu} + T_{\beta\mu\alpha} - T_{\mu\alpha\beta}. \quad (3)$$

In anholonomy basis, the torsion tensor becomes

$$T_{\hat{a}\hat{b}}^{\hat{c}} = \frac{1}{2}(\Gamma_{\hat{a}\hat{b}}^{\hat{c}} - \Gamma_{\hat{b}\hat{a}}^{\hat{c}}) + f_{\hat{a}\hat{b}}^{\hat{c}} \quad (4)$$

$$f_{\hat{a}\hat{b}}^{\hat{c}} = \frac{1}{2}e_{\hat{a}}^{\mu}e_{\hat{b}}^{\nu}(\partial_{\mu}e_{\nu}^{\hat{c}} - \partial_{\nu}e_{\mu}^{\hat{c}}), \quad (5)$$

where $f_{\hat{a}\hat{b}}^{\hat{c}}$ is anholonomy and $e_{\hat{a}}^{\mu}$ is tetrad frame. Torsion tensor can be decomposed into three irreducible parts [7]

$$T_{\mu\nu}^{\lambda} = \tilde{T}_{\mu\nu}^{\lambda} + \frac{2}{3}\delta_{[\nu}^{\lambda}T_{\mu]} + \frac{1}{3}\varepsilon_{\mu\nu\beta}^{\lambda}\hat{T}^{\beta}, \quad (6)$$

where

$$T_{\mu} = T_{\mu\lambda}^{\lambda} \quad (7)$$

is the trace part of torsion tensor(V). It has no relation to the spin density but will preserve the minimal coupling

and gauge invariance of electromagnetic field in ECSK theory [8].

$$\hat{T}^{\alpha} = \frac{1}{3!}\varepsilon^{\alpha}_{\mu\nu\lambda}T^{\mu\nu\lambda} \quad (8)$$

is the axial torsion vector. The last term of Eq.(6) is totally anti-symmetric part of torsion tensor(A) which is believed to couple with Dirac particles [7], and $\tilde{T}_{\mu\nu}^{\lambda}$ is the rest of traceless part but not totally anti-symmetric(TL). One defined the modified torsion tensor

$$Q_{\lambda\mu}^{\nu} = T_{\lambda\mu}^{\nu} + \delta_{\lambda}^{\nu}T_{\mu} - \delta_{\mu}^{\nu}T_{\lambda}. \quad (9)$$

In V4, the Levi-Civita connection is determined by metric compatible and torsion free condition. In U4, the torsion free condition is eliminated, so there is no unique solution of metric compatible connection except some proper constrained conditions are chosen.

In ECSK theory, the Einstein-Cartan-Sciama-Kibble equations are given by

$$\begin{aligned} G_{\mu\nu} &= k\theta_{\mu\nu} \\ Q_{\lambda\mu\nu} &= kS_{\lambda\mu\nu}, \end{aligned} \quad (10)$$

where $G_{\mu\nu}$ is the Einstein tensor with respect to U4 connection.

$$\theta^{\mu\nu} = T^{\mu\nu} + \nabla^*_{\alpha}(S^{\mu\nu\alpha} - S^{\nu\alpha\mu} + S^{\alpha\mu\nu}) \quad (11)$$

is the energy momentum tensor $T_{\mu\nu}$ plus extra spin contributions where $S^{\alpha\mu\nu}$ is spin density tensor and $\nabla^*_{\alpha} = \nabla_{\alpha} + 2T_{\alpha}$ where ∇_{α} is covariant derivative with respect to U4 connection.

In the framework of the early ECSK theory, they introduced a Weyssenhoff fluid to model a spinning perfect fluid as a source for the torsion [9]. Ray and Smalley provided a self-consistent variational principle for a Weyssenhoff spinning fluid in V4 and U4 [10, 11], where the spinning fluid is given by

$$\begin{aligned} T_{\mu\nu} &= T_{\mu\nu}(fluid) + T_{\mu\nu}(spin) \\ T_{\mu\nu}(fluid) &= (\varepsilon + p)U_{\mu}U_{\nu} + pg_{\mu\nu} \\ T_{\mu\nu}(spin) &= 2U_{(\mu}S_{\nu)\sigma}\dot{U}^{\sigma} + \nabla^*_{\sigma}[U_{(\mu}S_{\nu)}^{\sigma} - \tilde{\omega}_{\sigma(\mu}S_{\nu)}^{\sigma}]. \end{aligned} \quad (12)$$

where ε is the energy density, p is the pressure, U is four velocity, $\dot{U}^{\sigma} = U^{\alpha}\nabla_{\alpha}U^{\sigma}$; and $\tilde{\omega}_{\mu\nu}$ is the angular velocity associated with the spin density which can be defined as

$$\tilde{\omega}_{\mu\nu} = U^{\sigma}\nabla_{\sigma}e_{\mu}^{\hat{a}} \cdot e_{\hat{a}\nu}. \quad (13)$$

It is equal to vorticity vector when the acceleration is zero. The spin tensor is defined as

$$S_{\mu\nu} = k(e_{\mu}^{\hat{1}}e_{\nu}^{\hat{2}} - e_{\nu}^{\hat{1}}e_{\mu}^{\hat{2}}), \quad (14)$$

it is constrained by the Frenkel condition

$$S_{\mu\nu}^{\sigma} = S_{\mu\nu}U^{\sigma}. \quad (15)$$

The model of Godel cosmology in Riemann-Cartan space-time was based on this source by Smalley [4].

In this paper, the tetrad field is denoted by \cdot . The Greek alphabet ($\mu, \nu, \rho, \dots = 0, 1, 2, 3$) are used to denote tensor indices, The Latin alphabet with hat ($\hat{a}, \hat{b}, \hat{c}, \dots = 0, 1, 2, 3$) will be used to denote frame indices. The Minkowski space-time metric tensor raises and lowers tetrad indices and is fixed by $\eta_{ab} = e_{\hat{a}\mu} e_{\hat{b}\nu} g^{\mu\nu} = (-+++)$.

C. Vorticity

In relativistic hydrodynamics, the gradient of the velocity field u is decomposed in the following way [2, 3].

$$\nabla_\mu u_\nu = \frac{1}{3}\theta h_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu} + u_\mu a_\nu, \quad (16)$$

where $h_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$ is space-projection tensor.

$$\begin{aligned} \theta &= \nabla_\alpha u^\alpha \\ \sigma_{\mu\nu} &= h_\mu^\alpha h_\nu^\beta (\nabla_{(\alpha} u_{\beta)} - \frac{1}{3}\theta h_{\alpha\beta}) \\ a_\nu &= u^\alpha \nabla_\alpha u_\nu \end{aligned} \quad (17)$$

which are expansion scalar, shear tensor, and acceleration vector, respectively. The vorticity tensor is defined as

$$\omega_{\mu\nu} = h_\mu^\alpha h_\nu^\beta \nabla_{[\alpha} u_{\beta]}. \quad (18)$$

∇_α is the covariant derivative in U4, but all the equations have the same form in V4. The vorticity vector in V4 is given by

$$\omega^\alpha = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} u_\beta \omega_{\mu\nu} = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} u_\beta u_{\mu;\nu} = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} u_\beta u_{\mu,\nu} \quad (19)$$

where the semicolon and comma is covariant and partial derivative.

Vorticity geometrically measures the twisting of the congruence [12]. The meaning of vorticity vector is the rotation of connecting vector relates to Fermi-Walker frame. If a space-time with a time-like Killing vector ξ is considered, the four velocity of observer is $u^\alpha = \xi^\alpha / \sqrt{-\xi^\beta \xi_\beta}$. Each observer arranges his spatial basis vectors so that they connect to the same neighboring observers for all time t , which means the spatial basis is Lie-dragged along u . Lie-dragged locks the spatial frame to connecting vectors. That means the vorticity ω is equal to the rotation relative to Fermi-Walker frame which is physically realized by the system of gyroscopes. Therefore the angular velocity of the gyroscopic precession relative to the reference frame is given by $-\omega$ [2, 12].

From Eq.(19), we can see there is no difference between the vorticity vectors in V4 and flat space-time. It seems that V4 space-time cannot reflect the geometric property of vorticity vector unless the torsion tensor exists. The

vorticity vector with torsion in U4 is

$$\begin{aligned} \omega^\alpha_{(T)} &= \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} u_\beta \nabla_\nu u_\mu \\ &= \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} u_\beta [\partial_\nu u_\mu - (\tilde{\Gamma}_{\nu\mu}^\gamma - K_{\nu\mu}^\gamma) u_\gamma] \\ &= \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} u_\beta (u_{\mu;\nu} + K_{\nu\mu}^\gamma u_\gamma) \end{aligned} \quad (20)$$

where $\Gamma_{\nu\mu}^\gamma$ and $\tilde{\Gamma}_{\nu\mu}^\gamma$ are the connections of U4 and V4.

III. CALCULATION OF VORTICITY VECTOR AND SPIN-CURVATURE FORCE

A. Godel metric

The result derived by Smalley [4] could also be calculated alternately using the simple method which is to calculate Eq.(20) directly. The Godel metric is [13]

$$ds^2 = a^2[-(dt + e^x dy)^2 + dx^2 + \frac{1}{2}e^{2x} dy^2 + dz^2]. \quad (21)$$

The tetrad is chosen as

$$\begin{aligned} e_{\hat{t}} &= \frac{-1}{a}\partial_t, e_{\hat{x}} = \frac{1}{a}\partial_x, e_{\hat{y}} = \frac{-\sqrt{2}}{a}(\partial_t - e^{-x}\partial_y), e_{\hat{z}} = \frac{1}{a}\partial_z \\ e^{\hat{t}} &= -adt - ae^x dt dy, e^{\hat{x}} = adx, e^{\hat{y}} = \left(\frac{ae^x}{\sqrt{2}}\right)dy, e^{\hat{z}} = adz. \end{aligned} \quad (22)$$

The four velocity is defined as $u^\beta = e_{\hat{a}}^\beta$, so

$$u^\beta = \left(-\frac{1}{a}, 0, 0, 0\right). \quad (23)$$

The vorticity vector in V4 is given by Eq.(19), the result is

$$\omega^\mu = (0, 0, 0, -\frac{1}{2}\frac{\sqrt{2}}{a^2}). \quad (24)$$

Now, we consider the case in U4. The treatment by Smalley [4] to decide torsion tensor was based on two assumptions. First, the torsion tensor in anholonomy basis is proportional to anholonomy

$$T_{\hat{a}\hat{b}}^{\hat{c}} = \frac{1}{2}(\Gamma_{\hat{a}\hat{b}}^{\hat{c}} - \Gamma_{\hat{b}\hat{a}}^{\hat{c}}) + f_{\hat{a}\hat{b}}^{\hat{c}} = C \cdot f_{\hat{a}\hat{b}}^{\hat{c}}, \quad (25)$$

where C is constant. Second, the trace part of torsion tensor Eq.(7) will vanish because the spin fluid will not couple to that part. The anholonomy of the tetrad is

$$\begin{aligned} f_{\hat{y}\hat{x}}^{\hat{t}} &= -f_{\hat{x}\hat{y}}^{\hat{t}} = \frac{\sqrt{2}}{2a} \\ f_{\hat{x}\hat{y}}^{\hat{y}} &= -f_{\hat{y}\hat{x}}^{\hat{y}} = \frac{1}{2a} \end{aligned} \quad (26)$$

According to his model, there is a non-vanishing component of spin tensor $S_{\hat{x}\hat{y}}$, thus $C \cdot S_{\hat{x}\hat{y}}^{\hat{t}} = C \cdot S_{\hat{x}\hat{y}}^{\hat{t}} u^{\hat{t}} =$

$T_{\hat{x}\hat{y}}^{\hat{t}} \neq 0$, but $T_{\hat{x}\hat{y}}^{\hat{y}} = 0$ due to $u^{\hat{y}} = 0$. In order to make it consistent, he set $T_{\hat{x}\hat{y}}^{\hat{y}} = 0$, although $C \cdot f_{\hat{x}\hat{y}}^{\hat{y}}$ is not zero. This assumption will zero not only the trace part of torsion but also the traceless part of torsion which is not totally anti-symmetry. The only non-vanishing torsion tensor now is given by

$$T_{\hat{y}\hat{x}}^{\hat{t}} = C f_{\hat{y}\hat{x}}^{\hat{t}} = -T_{\hat{x}\hat{y}}^{\hat{t}} = C \frac{\sqrt{2}}{2a}. \quad (27)$$

It is transformed to holonomic basis

$$T_{yx}^t = -T_{xy}^t = \frac{-C e^x}{2}. \quad (28)$$

Using Eq.(20), the vorticity vector with torsion is

$$\omega^\mu_{(T)} = (0, 0, 0, \frac{1}{2} \frac{(-1 + C)\sqrt{2}}{a^2}). \quad (29)$$

When $C = 0$, torsion=0, Eq.(29) is identical to the vorticity vector in V4. When $C=2$,

$$\omega^\mu = -\omega^\mu_{(T)}, \quad (30)$$

which is the same as the result of Smalley [4]. It should be mentioned when $C=2$, the spinning fluid model of Ray and Smalley is consistent with the Godel cosmology in U4.

B. Kerr metric

In BoyerLinqvist coordinates, the Kerr metric takes the form

$$\begin{aligned} ds^2 = & -(1 - \frac{2mr}{\Sigma})dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \\ & - \frac{4mr \sin^2 \theta}{\Sigma} d\phi dt + (r^2 + a^2 + \frac{2mra^2 \sin^2 \theta}{\Sigma}) \sin^2 \theta d\phi^2 \\ \Sigma = & r^2 + a^2 \cos^2 \theta \quad \Delta = r^2 - 2mr + a^2 \end{aligned} \quad (31)$$

The local static frame [14] is given by

$$\begin{aligned} e_{\hat{t}} = & \sqrt{\frac{\Sigma}{\Sigma - 2mr}} \partial_t, e_{\hat{r}} = \sqrt{\frac{\Delta}{\Sigma}} \partial_r, e_{\hat{\theta}} = \frac{1}{\sqrt{\Sigma}} \partial_\theta, \\ e_{\hat{\phi}} = & \frac{-2mar \sin \theta}{\sqrt{\Delta \Sigma (\Sigma - 2mr)}} \partial_t + \frac{\sqrt{\Sigma - 2mr}}{\sin \theta \sqrt{\Delta \Sigma}} \partial_\theta \\ e_{\hat{t}} = & \sqrt{\frac{\Sigma}{\Sigma - 2mr}} \left(\frac{\Sigma - 2mr}{\Sigma} dt + \frac{2mar \sin^2 \theta}{\Sigma} d\phi \right), \\ e_{\hat{r}} = & \sqrt{\frac{\Sigma}{\Delta}} dr, e_{\hat{\theta}} = \sqrt{\Sigma} d\theta, e_{\hat{\phi}} = \frac{\Sigma \Delta \sin \theta}{\sqrt{\Delta \Sigma (\Sigma - 2mr)}} d\phi \end{aligned} \quad (32)$$

The four velocity is

$$u^\alpha = \left(\sqrt{\frac{\Sigma}{\Sigma - 2mr}}, 0, 0, 0 \right), \quad (33)$$

then the non-zero components of vorticity vector in V4 are

$$\begin{aligned} \omega^r = & -\frac{2mra \cos \theta (r^2 - 2mr + a^2)}{(r^2 + a^2 \cos^2 \theta)^2 (r^2 - 2mr + a^2 \cos^2 \theta)} \\ \omega^\theta = & -\frac{ma \sin \theta (r^2 - a^2 \cos^2 \theta)}{(r^2 + a^2 \cos^2 \theta)^2 (r^2 - 2mr + a^2 \cos^2 \theta)} \end{aligned} \quad (34)$$

In U4, now we will slightly modify the torsion used in Godel metric to the whole traceless part of torsion, thus

$$T_{\mu\nu}{}^\lambda{}_{(A+TL)} = T_{\mu\nu}{}^\lambda - \frac{2}{3} \delta_{[\nu}^\lambda T_{\mu]}. \quad (35)$$

Vorticity vector with torsion turns to

$$\begin{aligned} \omega^r_{(T)} = & \frac{(C - 1)2mra \cos \theta (r^2 - 2mr + a^2)}{(r^2 + a^2 \cos^2 \theta)^2 (r^2 - 2mr + a^2 \cos^2 \theta)} \\ \omega^\theta_{(T)} = & \frac{(C - 1)ma \sin \theta (r^2 - a^2 \cos^2 \theta)}{(r^2 + a^2 \cos^2 \theta)^2 (r^2 - 2mr + a^2 \cos^2 \theta)} \end{aligned} \quad (36)$$

When $C = 0$, torsion=0, we get the vorticity vector in V4. When $C=2$, $\omega^\alpha_{(T)} = -\omega^\alpha$.

The next problem needs to be considered is the meaning of vorticity vector of static observer in Kerr space-time. Considering the case at the equator $\theta = \pi/2$,

$$\omega^\alpha = (0, 0, \frac{-ma}{r^3(r - 2m)}, 0), \quad (37)$$

and the scalar vorticity

$$\Omega = (\omega^\alpha \omega_\alpha)^{1/2} = \frac{ma}{r^2(r - 2m)} = \frac{ma}{r^3} \left(1 - \frac{2m}{r}\right)^{-1} \quad (38)$$

is the precession of gyroscope due to frame dragging at equator which is the same as the result of Iyer and Vishveshwara calculated by Frenet-Serret method [12]. This is because we choose static observers(see Fig I.) which is proportional to time-like killing vector, whose bases frame vectors are lie-dragged along u,

$$\begin{aligned} P(u)L_u e_i &= 0 \\ P(u) &= h^\alpha{}_\beta \end{aligned} \quad (39)$$

It locks the spatial frame to the connecting vector of the killing congruence are always pointed to the same fixed stars [15, 16].

C. Lewis metric

The general stationary axi-symmetric metric considered is the standard form of the rotating metric [17].

$$ds^2 = -f dt^2 - 2k dt d\phi + l d\phi^2 + e^b (dr^2 + dz^2) \quad (40)$$

with coordinates (t, r, z, ϕ) . The metric potentials are only functions of r . The local static frame is given by [18]

$$\begin{aligned} e_{\hat{t}} &= 1/\sqrt{f(r)}\partial_t, e_{\hat{r}} = e^{-b(r)/2}\partial_r, e_{\hat{z}} = e^{-b(r)/2}\partial_z, \\ e_{\hat{\phi}} &= \frac{1}{\sqrt{f(r)l(r) + k(r)^2}}\left(\frac{-k(r)}{\sqrt{f(r)}}\partial_t + \sqrt{f(r)}\partial_\phi\right), \\ e^{\hat{t}} &= \sqrt{f(r)}dt + \frac{k(r)}{\sqrt{f(r)}}d\phi, e^{\hat{r}} = e^{b(r)/2}dr, \\ e^{\hat{z}} &= e^{b(r)/2}dz, e^{\hat{\phi}} = \frac{\sqrt{f(r)l(r) + k(r)^2}}{\sqrt{f(r)}}d\phi. \end{aligned} \quad (41)$$

The four velocity

$$u^\alpha = \left(\frac{1}{\sqrt{f(r)}}, 0, 0, 0\right). \quad (42)$$

Vorticity vector in V4 is

$$\omega^\alpha = (0, 0, \frac{1}{2} \frac{[f(r)k'(r) - k(r)f'(r)]}{e^{b(r)}f(r)\sqrt{f(r)l(r) + k(r)^2}}, 0); \quad (43)$$

and the vorticity vector in U4 with traceless part of torsion becomes

$$\omega^\mu_{(T)} = (0, 0, \frac{1}{2} \frac{(1-C)[f(r)k'(r) - k(r)f'(r)]}{e^{b(r)}f(r)\sqrt{f(r)l(r) + k(r)^2}}, 0). \quad (44)$$

When $C=0$, torsion=0, Eq.(44) is the vorticity vector in V4. When $C=2$, $\omega^\alpha_{(T)} = -\omega^\alpha$. We still want to know the meaning of vorticity vector belong to this observer. The scalar vorticity

$$\Omega = (\omega^i \omega_i)^{1/2} = \frac{e^{-b(r)/2} |k(r)f'(r) - f(r)k'(r)|}{2f(r)\sqrt{f(r)l(r) + k(r)^2}} \quad (45)$$

is the precession angular velocity of gyroscope due to frame dragging in Lewis space-time which is the same as the result of Herrera [19].

D. Schwarzschild metric

The well-known Schwarzschild metric is given by

$$ds^2 = -(1 - \frac{2m}{r})dt^2 + \frac{1}{1 - \frac{2m}{r}}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2. \quad (46)$$

The four velocity of static observer in Schwarzschild metric is hypersurface orthogonal that means the vorticity is zero. Therefore, we have to choose such an observer who has non-zero and physically meaningful vorticity. According to de Felice and Usseglio-Tomasset [20], the

tetrad was chosen as

$$\begin{aligned} e_{\hat{t}} &= \frac{1}{r\sqrt{\Lambda - w^2\sin^2\theta}}(\partial_t + w\partial_\phi), e_{\hat{r}} = \Sigma\partial_r, e_{\hat{\theta}} = 1/r, \\ e_{\hat{\phi}} &= \frac{1}{\sqrt{\Lambda - w^2\sin^2\theta}}\left(\frac{w\sin\theta}{\Sigma}\partial_t + \frac{\Sigma}{r^2\sin\theta}\partial_\phi\right), \\ e^{\hat{t}} &= \frac{\sqrt{r^3}}{\sqrt{r - 2m - w^2\sin^2\theta}}\left(\frac{r - 2m}{r^2}dt + -w\sin^2\theta d\phi\right), \\ e^{\hat{r}} &= -\frac{r\Sigma}{-r + 2m}dr, e^{\hat{\theta}} = rd\theta, \\ e^{\hat{\phi}} &= \frac{\sqrt{r^3}\sin\theta}{\sqrt{r - 2m - w^2\sin^2\theta}}\left(\frac{(-r + 2m)w}{r\Sigma}dt + \Sigma d\phi\right). \end{aligned} \quad (47)$$

where $\Lambda = \frac{1-2m/r}{r^2}$; $\Sigma = \sqrt{1 - \frac{2m}{r}}$; w is the orbital angular velocity. The four velocity is

$$u^\alpha = \left(\frac{1}{r\sqrt{\Lambda - w^2\sin^2\theta}}, 0, 0, \frac{w}{r\sqrt{\Lambda - w^2\sin^2\theta}}\right). \quad (48)$$

Then the non-zero components of vorticity vector in V4 are

$$\begin{aligned} \omega^r &= \frac{(-r + 2m)w \cos\theta}{-r + 2m + w^2r^3\sin^2\theta} \\ \omega^\theta &= -\frac{(-r + 3m)w \sin\theta}{r(-r + 2m + w^2r^3\sin^2\theta)}. \end{aligned} \quad (49)$$

Considering the case at the equator $\theta = \pi/2$

$$\omega^\mu = (0, 0, \frac{w(-r + 3m)}{r(-r + 2m + w^2r^3)}, 0), \quad (50)$$

$$\Omega = (\omega_\mu \omega^\mu)^{1/2} = \frac{w(-r + 3m)}{-r + 2m + w^2r^3}, \quad (51)$$

which is the gyroscopic precession at the equator [12]. Along a geodesic trajectory acceleration is zero, that leads to the Keplerian frequency $w^2 = m/r^3$, thus $\Omega = w$ which is the same as the result of Rindler and Perlick [21]. It brings out the de Sitter precession due to space-time curvature.

In U4, the vorticity vector with traceless torsion is

$$\begin{aligned} \omega^r_{(T)} &= -\frac{(-1 + C)(-r + 2m)w \cos\theta}{-r + 2m + w^2r^3\sin^2\theta} \\ \omega^\theta_{(T)} &= \frac{(-1 + C)(-r + 3m)w \sin\theta}{r(-r + 2m + w^2r^3\sin^2\theta)}. \end{aligned} \quad (52)$$

When $C=0$, torsion=0, the vorticity vector in V4 is obtained. When $C=2$, the vorticity changes to opposite sign.

E. Minkowski metric in Rotating coordinates

The transform $d\phi \rightarrow d\phi + \omega dt$ brings the Minkowski metric into the rotating form with coordinates (t, r, ϕ, z)

[21].

$$ds^2 = -(1 - w^2 r^2)[dt - \Sigma(r)d\phi]^2 + dr^2 + \frac{r^2}{1 - w^2 r^2}d\phi^2 + dz^2 \quad (53)$$

where $\Sigma(r) = \frac{r^2 w}{1 - w^2 r^2}$. The frame of static observer is

$$\begin{aligned} e_{\hat{t}} &= \frac{1}{\sqrt{1 - w^2 r^2}}\partial_t, e_{\hat{r}} = \partial_r, \\ e_{\hat{\phi}} &= \frac{wr}{c\sqrt{1 - w^2 r^2}}\partial_t + \frac{\sqrt{1 - w^2 r^2}}{r}\partial_{\phi}, e_{\hat{z}} = \partial_z, \\ e^{\hat{t}} &= \frac{1 - w^2 r^2}{\sqrt{1 - w^2 r^2}}dt - \frac{wr^2}{\sqrt{1 - w^2 r^2}}d\phi, \\ e^{\hat{r}} &= dr, e^{\hat{\phi}} = \frac{r}{\sqrt{1 - w^2 r^2}}d\phi, e^{\hat{z}} = dz \end{aligned} \quad (54)$$

The four velocity

$$u^\alpha = \left(\frac{1}{\sqrt{1 - w^2 r^2}}, 0, 0, 0\right). \quad (55)$$

The vorticity vector in V4 is

$$\omega^\alpha = (0, 0, 0, \frac{w}{1 - w^2 r^2}). \quad (56)$$

The vorticity scalar

$$\Omega = (\omega_\nu \omega^\nu)^{1/2} = \frac{w}{1 - w^2 r^2}, \quad (57)$$

which is the gyroscope precession in Minkowski space-time, or the Thomas precession angular velocity [21]. Again, in U4, the vorticity vector with traceless torsion becomes

$$\omega^\mu_{(T)} = (0, 0, 0, -\frac{(-1 + C)w}{1 - w^2 r^2}). \quad (58)$$

When $C=0$, torsion=0, the result is the vorticity in V4. When $C=2$, the vorticity vector changes to opposite sign.

F. Other parts of torsion and spin-rotation coupling

The traceless part of torsion tensor can be further decomposed into the totally anti-symmetric part (A) and the rest part (TL). The totally anti-symmetric part (A) is given by

$$T^{(A)}_{\mu\nu\sigma} = T_{[\mu\nu\sigma]} \quad (59)$$

and the rest part (TL) is

$$\tilde{T}^\lambda_{\mu\nu} = T_{\mu\nu}{}^\lambda - \frac{2}{3}\delta^\lambda_{[\nu}T_{\mu]} - \frac{1}{3}\varepsilon_{\mu\nu\beta}{}^\lambda \hat{T}^\beta. \quad (60)$$

Calculating the vorticity vector in all the previous cases under other parts of torsion tensor, the results are listed

in Table. I.

Table. I: The value of torsion constant C which turns the vorticity to opposite sign.

	Mink.	Schwarz.	Godel	Kerr	Lewis
A+TL	C=2	C=2	C=2	C=2	C=2
A	C=6	C=6	C=6	C=6	C=6
TL	C=3	C=3	C=3	C=3	C=3
V	X	X	X	X	X
A+TL+VC=2		C=2	C=2	C=2	C=2

X: no Influence on vorticity, A: traceless part and totally anti-symmetric, TL: traceless part but not totally anti-symmetric, V: trace part

In the case of traceless torsion (A+TL), (A), and (TL), the torsion constant $C=2$, 6, and 3 results in the opposite vorticity, respectively. The trace part of torsion will not influence the vorticity, so the full torsion has the same constant $C=2$ as in the A+TL case which can induce minus vorticity.

In order to understand the deeper physical meaning of the opposite sign of vorticity in U4, we calculate the axial torsion vector in all the above mentioned space-time, they all have the same form

$$\hat{T}^\alpha = \frac{C}{3}\omega^\alpha = \frac{C}{3}\Omega^\alpha_{(ob-gyro)}, \quad (61)$$

Ω is the angular velocity, (ob-gyro) means the quantity of observer relates to the gyroscope. It had been shown that the spin precession of a Dirac particle in the external torsion field [22–24].

$$\frac{dS}{dt} = -3\hat{T}^\alpha \times S. \quad (62)$$

Substituting Eq.(61) into Eq.(62)

$$\frac{dS}{dt} = -3\hat{T}^\alpha \times S = -C\Omega_{(ob-gyro)} \times S \quad (63)$$

which has the same form as the spin-rotation coupling [25, 26]. This effect describes the rotating observers with angular velocity would associate a hamiltonian, where S is the intrinsic spin, then the equation of motion $\hbar dS/dt = i[H, S]$ coincides with the precession equation of classical spin vector $dS_i/dt = -\varepsilon_{ijk}\Omega_j S_k$. Recently, It was shown [27, 28] that the dynamics of classical and quantum spins in curved space-time are identical. It is frame independent within the order of Schiffs precession. According to our calculations, the presence of torsion will not change the magnitude of gyroscopic precession but result in opposite sign. It seems nature to interpret it as the precession of quantum spin with positive and negative helicity. The inclusion of torsion in classical treatment leads to very similar behavior analogous to the property of quantum spin. This phenomenon was shown in another method by Pasini [29] who considered using asymmetric connection.

Zhang and Beesham [30] recovered the spin-rotation coupling of Dirac particle in the teleparallel theory. In

fact, it belongs to the case of $C=1$. When $C=6$, only totally anti-symmetric part of torsion is involved, $dS/dt = -6\Omega \times S$ should include the spin-rotation coupling and the effect between Dirac particle and torsion field. When $C=2$, axial torsion still contribute to spin-rotation coupling, but there should be other effects from TL part of torsion. On the other hand, the trace part of torsion will not contribute to opposite vorticity, this might resort to that it has no relation to spin. In addition, the helicity flip of Dirac spin induced by axial torsion was studied by Capozziello et al [31].

It is important to note that the torsion effect has different property between rotating and non-rotating space-time. Spin-rotation coupling appears in the rotating reference frame. For non-rotating space-time, the torsion effect which is in the shape of spin-rotation coupling will vanish in non-rotating frame. For rotating space-time, the angular velocity $\Omega_{(ob-gyro)} = -\Omega_{fg}$ which is the angular velocity of the gyroscopic precession due to frame dragging effect. According to gravitational Larmor theorem [32], the gyroscope undergoes frame dragging precession Ω with respect to observers at infinity, the gravitomagnetic effects are locally equivalent to inertial effects in a frame rotating with frequency $-\Omega$. Thus the torsion effect which is spin-rotation coupling shaped becomes

$$dS/dt = -C\Omega_{(ob-gyro)} \times S = C\Omega_{fg} \times S. \quad (64)$$

The effect will not vanish as long as gravitomagnetic field: $H(u) = 2\Omega_{fg}$ [15, 16] exists. It will lead to the presence of spin-gravitomagnetism coupling term in hamiltonian and gravitational Stern-Gerlach force which will be discussed in next section.

The unique property of torsion in rotating space-time could be seen alternately through axial torsion vector. The axial torsion vector is a measure of the deviation from spherical symmetry [23], but it depends on the tetrad. For Schwarzschild metric, the frame of static observer reflects the spherical symmetry of space-time so the axial torsion vector is identical to zero. In our case, the axial torsion appears in Schwarzschild metric due to the choice of rotating reference frame. In contrast, for Kerr metric, the axial torsion vector reflects the rotation of the space-time. The existence of torsion effect is independent on the reference frame in rotating space-time.

G. Gravitational Stern-Gerlach force

Considering the spin-rotation coupling, the intrinsic spin S of a particle would couple to the gravitomagnetic field of a rotating source via the interaction hamiltonian

$$H = S \cdot \Omega_{fg}. \quad (65)$$

It follows that the particle is subject to a gravitational Stern-Gerlach force given by

$$F = -\nabla(S \cdot \Omega_{fg}). \quad (66)$$

A spinning particle falls differently in the gravitational field of the Earth with spin up and spin down. In the correspondence limit, this force can be deduced from the Mathisson-Papapetrou spin-curvature force [33, 34]. The motion of classical spinning particles in gravitational field is described by Mathisson-Papapetrou equations. There exists a spin-curvature force.

$$u^\nu \tilde{\nabla}_\nu P^\mu = -\frac{1}{2} \tilde{R}^\mu_{\alpha\beta\gamma} u^\alpha S^{\beta\gamma} = F^\mu. \quad (67)$$

$$P^\mu = mu^\mu - u^\alpha \tilde{\nabla}_\alpha S^{\mu\nu} u_\nu, \quad (68)$$

where P^μ is the total 4-momentum, m is the particles mass in the rest frame, and $S^{\mu\nu}$ is the anti-symmetric classical spin tensor. It was shown [35, 36] that the motion of quantum spin particle coming out from the semi-classical limit of a spinor field coincides with a classical spinning particle at the lowest order in the spin. They all have the same spin-curvature coupling equation as Eq.(67). In the presence of torsion, this force becomes [37]

$$F_{(T)}^\mu = u^\nu \nabla_\nu P^\mu = -T_{\alpha\beta}{}^\mu P^\alpha u^\beta - \frac{1}{2} R^\mu_{\alpha\beta\gamma} u^\alpha S^{\beta\gamma}. \quad (69)$$

First, we analyze the spin-curvature force of Godel space-time in V4. The components of spin curvature force are

$$F^\mu = \left(\frac{e^x S^{ty}}{2a}, \frac{-S^{tx} + e^x S^{xy}}{2a}, \frac{-S^{ty}}{2a}, 0 \right). \quad (70)$$

The left hand side of Eq.(67) is given by

$$\begin{aligned} P^\mu = \\ t : (-m + ae^x \dot{S}^{ty} - aS^{tx} + ae^x S^{xy})/a \\ x : -\dot{S}^{tx} + e^x \dot{S}^{xy} - e^x S^{ty}/2 \\ y : (-e^x \dot{S}^{ty} + S^{tx} - e^x S^{xy})/e^x \\ z : -\dot{S}^{tz} + e^x \dot{S}^{zy} \end{aligned} \quad (71)$$

$$\begin{aligned} u^\alpha \tilde{\nabla}_\alpha P^\mu = \\ t : (2\dot{m} + 4a\dot{S}^{tx} - 2ae^x \ddot{S}^{ty} - 4ae^x \dot{S}^{xy} + ae^x S^{ty})/2a^2 \\ x : -(2\ddot{S}^{tx} - 2e^x \dot{S}^{ty} + 2e^x \ddot{S}^{xy} + S^{tx} - e^x S^{xy})/2a \\ y : (2e^x \ddot{S}^{ty} - 4\dot{S}^{tx} + 4e^x \dot{S}^{xy} - e^x S^{ty})/2ae^x \\ z : \ddot{S}^{tz} - e^x \ddot{S}^{zy} \end{aligned} \quad (72)$$

where a dot denotes differentiation with respect to t . By equating the left hand side to the right hand side of Eq. (67), four Simultaneous equations are obtained

$$\begin{cases} -2e^x \ddot{S}^{ty} + 4\dot{S}^{tx} - 4e^x \dot{S}^{xy} = 0 \\ -2e^x \dot{S}^{ty} - 2\ddot{S}^{tx} + 2e^x \ddot{S}^{xy} = 0 \\ \ddot{S}^{tz} - e^x \ddot{S}^{zy} = 0 \\ \dot{m} = 0 \end{cases} \quad (73)$$

Using the Pirani supplementary condition [38].

$$S^{\mu\nu}u_\nu = 0, \quad (74)$$

which implies

$$\dot{m} = 0, S^{ty} = 0. \quad (75)$$

In Godel cosmology, it is reasonable to set $\dot{x} = 0$ [4]. In that case, the constrained conditions Eq.(73) reduce to

$$\begin{cases} \dot{S}^{tx} - e^x \dot{S}^{xy} = 0 \\ \ddot{S}^{tz} - e^x \ddot{S}^{zy} = 0 \end{cases}. \quad (76)$$

Now we consider full torsion in U4, the spin-curvature force becomes

$$\begin{aligned} F_{(T)}^t &= \frac{(-1+C)^2 e^x S^{ty}}{2a}, \\ F_{(T)}^x &= \frac{(-1+C)^2 (-S^{tx} + e^x S^{xy})}{2a}, \\ F_{(T)}^y &= \frac{-(-1+C)^2 S^{ty}}{2a}, F_{(T)}^z = 0. \end{aligned} \quad (77)$$

For C=0 and C=2, the force will not change sign. This is because the torsion operates twice on the left hand side. The 4-momentum in U4 changes to

$$\begin{aligned} P_{(T)}^\mu &= \\ t : & (-m + ae^x \dot{S}^{ty} + (-1+C)aS^{tx} - (-1+C)ae^x S^{xy})/a \\ x : & -\dot{S}^{tx} + e^x \dot{S}^{xy} + (-1+C)e^x S^{ty}/2 \\ y : & (-e^x \dot{S}^{ty} - (-1+C)(S^{tx} - e^x S^{xy}))/e^x \\ z : & -\dot{S}^{tz} + e^x \dot{S}^{zy} \end{aligned}. \quad (78)$$

Using the conditions Eq.(74)-(76), for C=2, the following result is derived

$$\begin{cases} u^\alpha \tilde{\nabla}_\alpha P^\mu = F^\mu \\ u^\alpha \nabla_\alpha P^\mu = -F^\mu \end{cases}. \quad (79)$$

In quantum physics, The opposite Stern-Gerlach force is relating to the spin with opposite helicity. It provides a good analogue to interpret the opposite sign of spin-curvature force as the behavior of different helicity of spin which supports our suggestion about opposite gyroscopic precession.

H. ECSK equations in vacuum solution

The vacuum solution in V4, for example, Schwarzschild and Kerr metric, their Einstein equations should take the form

$$\tilde{G}_{\mu\nu} = 0. \quad (80)$$

The Einstein tensor and Ricci curvature are zero due to the vacuum, but the four ranks Riemann curvature tensor is not zero because the curvature will propagate. The Einstein equation of vacuum solution in U4 becomes

$$\tilde{G}_{\mu\nu} + \text{Torsion term} = G_{\mu\nu} = k(T_{\mu\nu} + T_{\mu\nu}(\text{spin})), \quad (81)$$

where $\tilde{G}_{\mu\nu}$ is zero due to vacuum which is $T_{\mu\nu} = 0$. If $G_{\mu\nu} = 0$, the only possible solution is $T_{\mu\nu}(\text{spin}) = 0$. All the matters which can induce curvature and torsion are zero. This is the vacuum in U4.

First, we consider the traceless part of torsion (A+TL). The non-vanishing components of Einstein tensor of Schwarzschild metric in U4 are

$$\begin{aligned} G_{tt} &= C(3r^2 C \cos^2 \theta - 10r C \cos^2 \theta m + 3r^2 + 6Crm \\ &\quad + 18m^2 - 12rm - Cr^2 - 9Cm^2 - 3r^2 \cos^2 \theta - 18\cos^2 \theta m^2 \\ &\quad + 12r \cos^2 \theta m + C9\cos^2 \theta m^2)/9r^4(-1 + \cos \theta)(\cos \theta + 1), \\ G_{\phi t} &= Cw(-r^2 C \cos^2 \theta - 2r C \cos^2 \theta m + 9r \cos^2 \theta m + 3r^2 \\ &\quad - 15rm + 18m^2 - Cr^2 + 6rCm - 9Cm^2 - 18\cos^2 \theta m^2 \\ &\quad + 9C \cos^2 \theta m^2)/3r(r - 2m - w^2 r^3 + w^2 r^3 \cos^2 \theta), \\ G_{rr} &= C(r^2 C \cos^2 \theta - 14r C \cos^2 \theta m - 42rm + 54m^2 + 9r^2 \\ &\quad - 3Cr^2 + 18rCm - 27Cm^2 + 42r \cos^2 \theta m - 54\cos^2 \theta m^2 \\ &\quad - 9r^2 \cos^2 \theta + 27C \cos^2 \theta m^2)/9r^2(-r + 2m)^2(\cos^2 \theta - 1), \\ G_{\theta r} &= -C \cos \theta (3r - 6m - rC + 3Cm) \\ &\quad /9r \sin \theta (-r + 2m), \\ G_{r\theta} &= -C \cos \theta (12m - rC + 3Cm - 6r) \\ &\quad /9r \sin \theta (-r + 2m), \\ G_{\theta\theta} &= C(-r^2 C \cos^2 \theta + 6r^2 \cos^2 \theta - 21r \cos^2 \theta m \\ &\quad - 6r C \cos^2 \theta m + 9rm + 12Crm - 18Cm^2 - 18m^2 - 2Cr^2 \\ &\quad + 18C \cos^2 \theta m^2 + 18\cos^2 \theta m^2)/9r(-r + 2m) \sin^2 \theta, \\ G_{t\phi} &= Cw(r^2 C \cos^2 \theta - 10r C \cos^2 \theta m - 3rm + 12Crm \\ &\quad - 18Cm^2 + 9m^2 - 2Cr^2 + 3r \cos^2 \theta m + 18C \cos^2 \theta m^2 \\ &\quad - 9\cos^2 \theta m^2)/3r(r - 2m - w^2 r^3 + w^2 r^3 \cos^2 \theta), \\ G_{\phi\phi} &= C(3Cr^2 \cos^2 \theta - 14r C \cos^2 \theta m - 9r \cos^2 \theta m \\ &\quad + 21rm - 18m^2 - 6r^2 - 2Cr^2 + 12rCm - 18Cm^2 \\ &\quad + 18\cos^2 \theta m^2 + 18C \cos^2 \theta m^2)/9r(-r + 2m). \end{aligned} \quad (82)$$

All components are proportional to torsion constant C, when C=0, it will reduce to the vacuum solution in V4, thus Einstein tensor is zero.

Now we turn to calculate the case with full torsion (A+TL+V) in U4, The non-zero components of Einstein

tensor in U4 with full torsion are

$$\begin{aligned}
G_{tt} &= \frac{C(-1+C)(-r+2m)^2}{r^4}, \\
G_{t\phi} &= \frac{-C(-1+C)(-r+3m)m\omega\sin^2\theta}{r(r-2m-w^2r^3\sin^2\theta)}, \\
G_{rr} &= \frac{C(-1+C)}{r(-r+2m)}, \\
G_{\theta r} &= \frac{C(-1+C)\cos\theta}{r\sin\theta}, G_{\theta\theta} = \frac{C(-1+C)m}{r}, \\
G_{\phi t} &= \frac{C(-1+C)\omega(-r+2m)(r-3m\sin^2\theta)}{r(r-2m-w^2r^3\sin^2\theta)}, \\
G_{\phi\phi} &= \frac{C(-1+C)m\sin^2\theta}{r}.
\end{aligned} \tag{83}$$

When $C=2$, it will cause vorticity to opposite sign which could be interpreted as the spin flip, thus the space-time might have particle spin prior and induce space-time torsion. When $C=0$, torsion $=0$, and all the components of Einstein tensor are zero. We get the vacuum of Schwarzschild metric in V4 that is $\tilde{G}_{\mu\nu} = 0$. However, there is still another condition to make the vacuum in U4. For $C=1$, the Einstein tensor also vanishes, but torsion tensor still exists. As we discussed before, the vanishing Einstein tensor of vacuum solution in U4 arising from $T_{\mu\nu}(\text{spin}) = 0$. It implies the full torsion will propagate, but traceless part of torsion will not. The result actually obeys the principle of contact interaction of spin [6] in ESK theory and still resort to the reason that only traceless torsion couple to spin. The same result can also be obtained in Kerr metric which is the exterior vacuum solution of rotating body. We will not list the result due to its lengthy expression.

Using the identity of vorticity vector which can be found in Ref. [39]:

$$\tilde{\nabla}_\alpha \omega^\alpha = 2a_\alpha \omega^\alpha. \tag{84}$$

This identity will have the same form as that in U4 with respect to U4 connection [40]:

$$\nabla_\alpha \omega^\alpha_{(T)} = 2a_{\alpha(T)} \omega^\alpha_{(T)} \tag{85}$$

In the case of traceless part of torsion, when $C=2$, $\omega^\alpha_{(T)} = -\omega^\alpha$ then

$$-\tilde{\nabla}_\alpha \omega^\alpha = -2a_{\alpha(T)} \omega^\alpha \tag{86}$$

thus

$$a_{\alpha(T)} = a_\alpha. \tag{87}$$

If the observer moves from V4 to U4 space-time (A+TL torsion), he will feel the same acceleration but has chance (very low probability in weak field) to observe the helicity flip of co-moving spin.

Now we turn to consider the situation with full torsion, for $C=2$

$$\nabla_\alpha \omega^\alpha_{(T)} = -\tilde{\nabla}_\alpha \omega^\alpha + K_{\alpha\beta}{}^\alpha \omega^\beta = -2a_{(T)} \omega^\alpha \tag{88}$$

thus

$$2(a_{\alpha(T)} - a_\alpha) \omega^\alpha = -K_{\alpha\beta}{}^\alpha \omega^\beta. \tag{89}$$

The trace part of torsion contributes to the difference of acceleration that the observer feels between V4 and U4.

IV. DISCUSSIONS AND CONCLUSIONS

In the Godel space-time, the vorticity vector is the global rotation that means the whole galaxy with spinning fluid rotates with the same angular velocity. After calculating the vorticity vector of other rotating space-time, we eventually realize the angular velocity which can flip to opposite sign is the gyroscopic precession caused by frame dragging effect. In nonrotating space-time, Schwarzschild and Minkowski metric, the minus sign will also couple to the de Sitter and Thomas precession.

Smalley attributed the minus vorticity to the frame dragging effect that appears in the static space-time by adding spinning fluid [4]. If we image minus vorticity as opposite gyroscopic precession, for rotating metric, it means opposite gravitomagnetic field. It seems unreasonable when the rotating gravitational source is not changed. We suggest that the better interpretation is the dynamics with respect to the helicity of spin. The opposite gyroscopic precession could be imaged as the classical analogy of the precession of particle with positive and negative helicity.

Analyzing the Mathisson-Papapetrou equations of Godel Space-time, the torsion constant which can induce minus vorticity will also induce minus spin-curvature force. The Spin-curvature force is the gravitational Stern-Gerlach force in weak field limit. It is nature to link the force with opposite direction to the behavior of particles with spin up and down which is analogue to Stern-Gerlach force of quantum spin. It supports our previous suggestion and also consists with the results that the dynamics of classical and quantum spins in curved space-time are identical [27, 28]. In our purely classical treatment, the inclusion of torsion tensor results in very similar property of quantum spin should resort to ESK theory that spin is the source of torsion.

Although we did not assume the spin fluid energy momentum tensor prior like other models [4, 5], non-zero torsion might imply that intrinsic spins have already existed. The trace part of torsion cannot contribute to opposite sign of vorticity because it has no relation with particle spin. The whole traceless part will induce helicity flip, it includes very similar result of spin-rotation coupling caused by totally anti-symmetric torsion and other contribution from the rest part of traceless torsion. In non-rotating space-time, the torsion effect in the shape of spin-rotation coupling will vanish if the reference frame is non-rotating. In rotating space-time, the torsion effect in the shape of spin-gravitomagnetism coupling was obtained through the local static observer. This term will not vanish as

long as gravitomagnetic field exists in the space-time. It might imply the result of some authors [41, 42] that torsion also originates from macroscopic rotation, and we suggest the torsion effect is similar to the coupling of spin and macroscopic rotation. Moreover, we show the

full torsion will propagate in vacuum. It is demonstrated that the torsion trace part plays the essential role of propagating torsion and traceless part contributes to spin-spin contact interaction.

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